USING CSRBFS FOR SURFACE RETOUCHING

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Abstract

In this paper, we present a novel fast algorithm for surface retouching of geometric objects. A space-mapping technique is used to transform a missing (or damaged) part of a surface into a different shape in a continuous manner. Experimental results are included to demonstrate the feasibility of our approach. The proposed approach shows the obvious relationship between the surface retouching problem and image inpainting. We consider shape transformation as a general type of operation for restoring missing data, and attempt to approach the well-known problem of "fulfillment" of damaged or missing surface areas from a single point of view, namely, that of the space mapping technique.

Key Words: Radial Basis Functions, Space Mapping, Surface Retouching

1. Introduction

Many recent studies have focused on exploring shape transformation as a basic operation in computer graphics (CG) and computer-aided geometric design (CAGD). The operation involves transforming a given geometric shape into another in a continuous manner. Important examples of surface fitting and deformation have been intensively investigated in the past few years, and various strategies have been proposed to minimize the user interaction; however, the problem of fitting a surface to control points still remains a largely unsolved problem of great practical importance. In fact, if we want to use the idea of fitting a surface to control points as a design tool, it is important that our fitting method does what our intuition would expect. In this paper we show that compactly supported radial-basis functions (CSRBFs) [1] offer a mechanism for obtaining extrapolated points of a damaged surface, and, in fact, exhibit high-quality restoration results in surface-retouching examples. We consider shape transformation as a general type of operation for restoring missing data, and attempt to approach the well-known problem of “fulfillment” of damaged or missing surface areas from a single point of view, namely, that of a space mapping technique. One of the most attractive features of radial-basis functions (RBFs) is the simplicity of implementation, and researchers usually state that RBF methods guarantee automatic mesh repair and interpolation of large irregular holes. However, the work required for correct reconstruction of an object is nearly proportional to the total number of scattered data points. The amount of computation is thus significant, even for a moderate number of nodes. If we have a gap in our data set, we face the problem of evaluating the functions at extra point; if the radius of support is quite large, then the cycle associated with the matrix of linear equations will include nearly all the points of the input data. Moreover, reliable results in reconstruction cannot be achieved without a sufficiently uniform distribution of scattered data. To attain real-time processing of a huge amount of input data, we replace the reconstruction process by the space-mapping technique. This allows disconnected surface areas to be reconstructed by using a sufficiently small radius of support as we will demonstrate in various applications: surface recovering of missing elevation data and surface refinement of damaged data.

The approach proposed here originates from the obvious relationship between 3D surface retouching (or reconstruction of missing data) and the 2D inpainting problem. For more references, see the pioneering work of Bertalmio et al. [2], which describes an image-inpainting (non-texture) algorithm based on partial differential equations. See also the recent studies [3], [4], and [5].

The main idea of the approach is to use the user-defined portions of the input image to be retouched (see Figure 2). The algorithm treats the input image as three separate channels (R, G, and B) and fills in areas to be inpainted. Actually, it is an extrapolation problem in which information from outside a masked area is propagating. It is known that the 3D space-mapping technique can be applied to surface modification. Naturally, user-selected data sets which after local transformation are represented in the form $z = f(x,y)$, where the z coordinate is used as one channel can be used as control points. In practice, the technique requires that a masked surface represented by polyhedrons be a closed, oriented manifold embedded in 3-space and have the property that, around every one of its points, there exists a neighborhood that is homeomorphic to a plane. That is, we can deform the surface locally into the plane without tearing it or identifying separate points with each other. Thus the term "surface retouching" is used to denote the deformation of one polyhedron into another between the boundaries of two shapes.

We propose a fast practical algorithm for surface retouching, which we believe can generate further attention from the CG community. Let us point out here that we clearly understand limitations of the proposed technique We have to notice that the results of surface retouching (see Figure 3) depend on the user-defined points and that the picture is a continuation of an area surrounding the region to be retouched. Perhaps Figure 4 does not offer a detailed representation of the damaged area. Nevertheless, according to our experiments, stitching nodes of damaged areas of 3D polygonal models by using triangles will produce more visible artifacts, and standard-shape smoothing approaches are not appropriate for removing such small shape defects. All the same, although we demonstrate the applicability of the proposed technique by restoration of "elevation data," this technique is applied in 3D space and can also serve for sculpting or editing a surface according to the user demands.

The rest of the paper is organized as follows. The next section gives a short overview of shape transformation techniques and studies related to the surface retouching problem. We discuss the notion of a 3D warping technique and software algorithm in Section 3. Application examples of surface recovering are presented in Section 4. Section 5 concludes the paper.

2. Related Work

Mappings can be controlled by numerical parameters of predefined functions, by control points, and by differential equations. An overview of all transformation techniques is beyond the scope of this paper; here we present a short overview of papers concerning the problem.

Shape transformation is a useful tool for many applications such as computer animation, CAGD, forensic identification. Chen et al. [6] and Skaria et al. [7] present a very good overview of existing methods for shape reconstruction and modification. For more references, see [8].

A vast literature is devoted to the subject of scattered data interpolation. This can be used for space mapping and, if applied to some point set in the space, it changes that set into a different one. In spite of a flurry of activity in the field of scattered data reconstruction and interpolation, this problem remains difficult and computationally expensive. A very good overview of related studies, problems, and limitations could be found in Lee et al. [9], which addresses these problems and introduces a very fast algorithm for constructing C^2-continuous interpolation functions.

Shape reconstruction from given points can be thought of as a special case of transformation. One approach is to use methods of scattered data interpolation based on the minimum-energy properties [10], [11], [12]. These methods are widely discussed in the literature (see also [13] and [14]).

The benefits of modeling with the help of RBFs have been recognized in many studies. To the best of our knowledge, the first publications on using discrete 2D landmark points were that of Bookstein [15], [16]. RBFs were adapted for computer animation [17], medical applications [18], [15], and surface reconstruction [19], [20]. However, the required computational work is proportional to the number of grid nodes and the number of scattered data points. Special methods for reducing the processing time were developed for thin plate splines, and were discussed in [21], [22], and [23]. In spite of significant progress in the field of implementing RBFs and CSRBFs [24], [25] for reconstruction purposes, it is still an open question whether it is possible to handle realistic amounts of data in real time. We suppose that they are suitable for sufficiently moderate 3D data sets; for instance execution time is about 300 seconds for 36000 points, without time expenses for surface extraction as it was reported in [24]. Nevertheless, they possess many features that make them very attractive for CAGD applications dealing with modification of geometric objects.

Problems of reconstructing missing portions of geometric objects that appear in CAGD have been investigated in the past few years. Three approaches have been dominant in the CAGD area: the first one works with 3D polygonal models to stitch damaged or incorrectly calculated nodes of 3D geometric objects [26], the second one is an approach dealing with fitting of the data generated according to some geometrical features such as curvature (for more references, see [27]), and the third one is actually based on a well-founded mathematically set-level approach. Partial differential equations are widely used to model a surface subject to certain constraints (see [28] and [29]). Whitaker and Breen [30] give an exhaustive overview of this topic; their paper presents an improved numerical algorithm that solves particular problems in geometric modeling. A different way of reconstructing missing portions of geometric objects is to use a "cloning" approach, see for instance the work of Savchenko and Shmitt [31] where an application of 3D-space mapping technique and numerical optimization with a specially designed genetic algorithm to a problem concerning CAGD in dentistry was presented. But this approach has two main drawbacks. In the first place, a standard or "cloning" surface should be given. Second, human knowledge and experience is necessary to assign initial correspondence points to compensate for the gap between the region of restoration and the existing part of the object.

3. Algorithm

For three dimension arbitrary area Ω, the thin–plate interpolation is the variational solution that defines a linear operator T when using the following minimum condition :

$$\int_{\Omega} \sum_{|\alpha|=m} m!/\alpha!(D^{\alpha}f)^2 d\Omega \rightarrow min,$$

where m is a parameter of the variational functional and α is a multi-index. It is equivalent to using the radial basis functions $\phi(r) = r^1$ or r^3 for $m = 2$ and 3 respectively, where r is a Euclidean distance between two points. Since the function $\phi(r)$ is not compactly supported, the corresponding system of linear algebraic equations (SLAE) is not sparse or bounded. Storing the lower triangle matrix requires $O(N^2)$ real numbers, and the computational complexity of a matrix factorization is $O(N^3)$. Thus, the amount of computation becomes significant, even for a moderate number of points.

Wendland in [1] constructed a new class of positive definite and compactly supported radial functions for *1D*, *3D* and *5D* spaces of the form

$$\phi(r) = \begin{Bmatrix} P(r) \; 0 \le r \le 1 \\ 0 \quad\; r > 1 \end{Bmatrix}$$

whose radius of support is equal to 1. $\phi(r) = (1 - r)^2$, which is an interpolated function that supports only C^0 continuity, is used. However, other functions that support a higher continuity can be applied. An investigation [32] of the smoothness of this family of polynomial basis functions shows that each member $\phi(r)$ possesses an even number of continuous derivatives.

The volume spline $f(P)$ having values hi at N points P_i is the function

$$f(P) = \sum_{j=1}^{N} \lambda_j \phi(|P - P_j|) + p(P), \qquad (*)$$

where $p = v_0 + v_1 x + v_2 y + v_3 z$ is a degree one polynomial. To solve for the weights λ_j we have to satisfy the constraints hi by substituting the right part of equation (*), which gives

$$h_i = \sum_{j=1}^{N} \lambda_j \phi(|P_i - P_j|) + p(P_i),$$

Solving for the weights λ_j and v_0, v_1, v_2, v_3 it follows that in the most common case there is a doubly bordered matrix ***T***, which consist of three blocks, square sub-matrices ***A*** and ***D*** of size $N\times N$ and 4×4 respectively, and B, which is not necessarily square and has the size $N \times 4$.

We employed an approach that uses displacements of N control points as a difference between an initial and final geometric form. The approach is illustrated by local deformations of data points homeomorphic to a plane; nevertheless, it is important to reiterate here that we formulate the surface-retouching problem in terms of the space-mapping technique, where transformations in *x,y,z* directions are allowed. Also, heights hi, are not necessarily arbitrary points of Euclidian space $\mathbf{E}^n$. Our approach gives an attractive possibility of using function mapping for controlling local deformations by placing arbitrary control points inside or outside an initial implicitly defined object **G**, and they are assumed to belong to the surface of a modified object $\mathbf{G}^m$. Thus, the control points define the deformation of G resulting in $\mathbf{G}^m$. In practice, it leads to the construction of a new defining function and the extraction of a new isosurface as it was discussed in [19]. In the paper we have used an algebraic sum of a sphere and a spline based on RBFs to reconstruct 3D solids (even non-convex solids) from a given set of scattered surface points. CSRBFs in combination with function mapping were used for surface reconstruction as it is discussed in [25].

Interpolation of (x,y,z) points is implemented in $\mathbf{R}^3$ and defines a relationship between coordinates of points in the original and deformed objects. Landmarks situated in the mask area Ω^+ defined by the user (Figure 3(b), grey square) naturally do not belong to the area Ω^R to be retouched (Figure 3(a)). The difference between Ω^+ and Ω^R defines the landmark area Ω^L. The inverse mapping function that interpolates the z-heights and is needed to calculate reconstructed (destination) points z d_i is given in the form:

$$z^s_i = f(P) + z^d_i,$$

where the components of the volume splines $f(P)$ interpolating displacements of starting points z^s_i are used to calculate points that belong to the area Ω^R.

In practice, the reconstruction operation consists of the following steps: data sorting for the landmarks area Ω^L, construction of an SLAE, solution of the SLAE, and evaluation of the functions for a region to be retouched. In fact, while the solution of the system is the limiting step, construction of the matrix and evaluation of the functions may also be computationally expensive.

1 Sorting scattered data

Our first goal is to build an octal tree data structure [33] from the original point data. For more references see, for example, [34]. Afterwards this tree is used to search for neighbors of any point among the given N points. The neighbors are points of a sphere of radius r whose origin is located at the given point. To make the sub-matrix ***A*** a band diagonal, we need to re-enumerate the initial points in a special way. An efficient approach based on the use of variable-depth octal trees for space subdivision was proposed in [25]. It allows us to obtain the resulting matrix as a band-diagonal matrix that reduces the computational complexity.

To store the band-diagonal matrix ***A***, we use a so-called profile form or a slightly modified Jennings envelope scheme [35]. An array can be used for diagonal elements; values of non-diagonal elements and corresponding indices of the first non-zero elements in the matrix lines are placed in two additional arrays. As a result of applying this algorithm, a band-diagonal sub-matrix is constructed as it is illustrated in Figure 1.

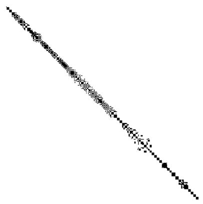


Figure 1. An example of typical sub-matrix $\boldsymbol{A}$.

2. SLAE solution

For a symmetric and positive definite matrix, a special factorization, called Cholesky decomposition, is about twice as fast as alternative methods for solving linear equations by Gaussian LU decomposition [36]. A combination of block Gauss solution and Cholesky decomposition was proposed by George and Liu in [37], and in our software tools we follow their proposal. After breaking up one linear set into a triangular set of equations, these equations can be solved by forward substitution and back substitution three times if we have three right-hand sides.

4. Experiments

Floating point arithmetic is used; thus, before processing the input data we prefer to normalize them in order to reduce the number of possible truncation errors. To illustrate the applicability of the space-mapping technique to the surface-retouching problem, we first show example of image inpainting.

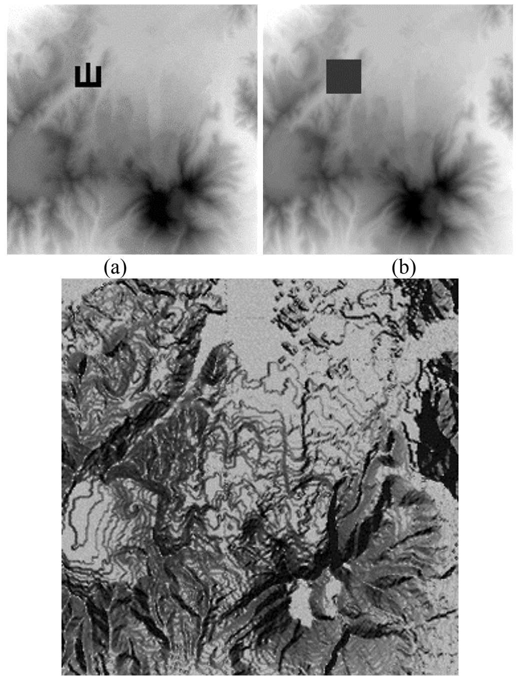
(a)

(b)

Figure 2. Example illustrating our approach.

Implementation of the above algorithm in an Adobe Photoshop plug-in (developed by our group and available for download from the Web [38]) allows us to reconstruct complicated images and indirectly (by visual inspection) has proved suitable restorations with a practical level of accuracy. A synthetically produced text area (see Figure 2 (a)) is the reconstruction area Ω^R. Adobe Photoshop® graphic user interface allows us to select automatically this area, after that a landmark area Ω^L as a slight extension of the Ω^R area (one or two pixels are sufficient) is automatically calculated. Figure 3(b) shows a result of image retouching.

Figure 3 illustrates the applicability of the space transformation technique to surface retouching of elevation data. Figure 3(a) presents the "image" of 300 × 300 elevation points given in a pixel raster, which shows elevation data (in fact, contour maps) of Bandai Mountain, a fragment of a volcano, in Japan.



(a) (b)

(c)

Figure 3. (a) "Image" of contour maps of Bandai Mountain, a fragment of a volcano. The **Ш** shape shows the surface Ω^R to be retouched. (b) The grey square shows the mask area Ω^+ defined by the user. (c) Result of surface retouching. Ray tracing of 300 × 300 elevation points. Image size: 300 × 300 points. Size of Ω^R area: 545 points. Size of Ω^L area: 1205 points. Selected radius: 0.03. Processing time (SGI Octane 300 Mhz): Data sorting - 0.1 sec; $\boldsymbol{LL^t}$ decomposition, forward and back substitution - 0.22 sec; Reconstruction time - 0.06 sec.

We use the term "image" in the sense that an image may be defined as a two-dimensional array of numbers. In addition, the numbers or elevation data may represent a visible image. Visual evaluation of the result is not sufficient for an appraisal of the algorithm. To estimate the accuracy of the restoration, we used the root mean square measure (RMS) of the error for the standard (not damaged by **Ш** shape) data as reference data and the terrain with missing data, Figure 3(a). Figure 3(c) shows an image of ray-traced elevation data as a result of restoration. The RMS is about one percent, which agrees with the accuracy of the elevation data.

There is a need for a simple and reliable method for surface retouching of polygonal surfaces. From our point of view, the most acceptable application of the proposed technique is automated correction of range data. One of the main drawbacks of range data acquisition with a laser scanner is that data may be missing or erroneous owing to specularities. For example, small spherical concavities on the "Moai" model (see Figure 4) appear at places where the emitted laser beam is orthogonal (or almost orthogonal) to the surface of the scanned model. Range data is captured by the Minolta VIVID 700 laser scanner. The scanner acquires 200 × 200 range images within an operating range of 0.6 to 2.5 m.

Standard shape-smoothing approaches are not appropriate for removing such small shape defects, because:
(1) usually they are applied to the whole model and therefore will over smooth (blur) curved regions without the defects, and
(2) they do not take account of the specific round shapes of the defects.

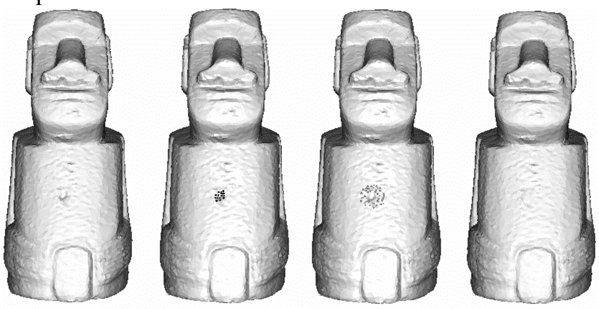


Figure 4. The left image shows the "Moai" model, whose surface was constructed from range data (courtesy of Dr. A. Belyaev of The University of Aizu). A spherical concavity can be observed (central area of the model). The right image is the same surface after retouching. Model size: 10002 vertices and 20000 polygons. Size of Ω^R area: 28 vertices. Size of Ω^L area: 57 vertices. Selected radius: 0.05. Processing time (AMD Athlon 1000 Mhz, 128 MB RAM, Microsoft Windows 2000): Data sorting - 0.001 sec; LL^t decomposition, forward and back substitution - 0.001 sec; Reconstruction time - 0.001 sec.

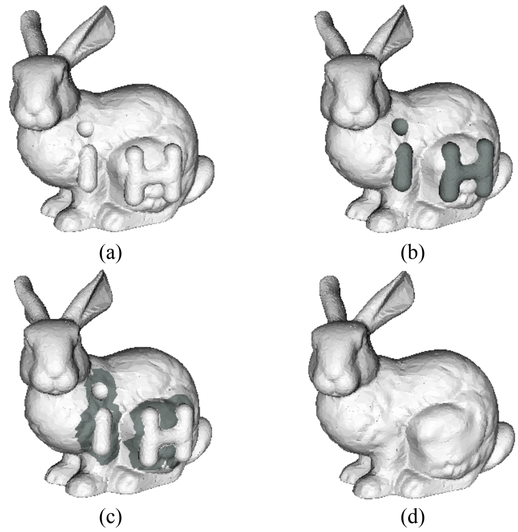


Figure 5. A modified "Stanford Bunny" [39] model (courtesy of Yutaka Ohtake of Max-Planck-Institut für Informatik). (a) Original (source) data: 10526 vertices, 20835 polygons. (b) Grey area: the area Ω^R to be retouched (930 vertices), (c) Grey area (height data), which defines the vertices used for interpolation (807 vertices). (d) The same surface after retouching. Processing time (AMD Athlon 1000 Mhz, 128 MB RAM, Microsoft Windows 2000): 1 sec. Compact matrix size: 1139180 bytes. Selected radius of support: $r = 0.2$.

Surface retouching was applied for a synthetic polygonal model shown in Figure 5 to illustrate an applicability of our approach for retouching of sufficiently large areas.

5. Conclusions

In this paper, we presented a simple and fast algorithm for surface retouching. To achieve the objectives of simplicity and speed, we replace the reconstruction process by a space mapping technique, which allows reconstruction of disconnected surface areas and allows further modifications (sculpting) according to selected by the user 3D vectors of deformations. Our approach shows the obvious relationship between the surface- and image-retouching problems. The method has the property that the mask Ω^+ need not include exactly all the regions to be retouched; it allows Ω^+ to be refined interactively in many steps to preserve the algorithm's speed, and even to improve the quality of restoration by assigning user-defined points in the area Ω^R.

We have found that CSRBFs produce good results in the sense of visual appearance, accuracy of restoration, and processing time. The algorithm is two to three orders of magnitude faster than methods based on partial differential equations.

We are going to use this approach to design an interactive system for CAGD applications (an integrated system is now under construction), and to apply it to volumetric data structures and sets of laser data.

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